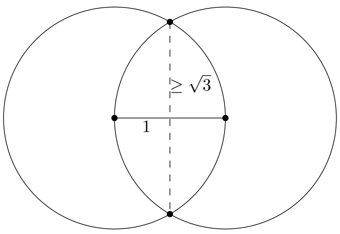
Graph Theory Problem Poetry

Problem 4 *(by Joachim Breitner)*
In the plane of Euklid, $\mathbb{R} \times \mathbb{R}$,
the set S be x points, spread out far:
between any two of which
the distance always is
either one, never less, maybe more.

Count pairs with distance 1, add six!
Show: That's no more than $3 \cdot x$.

Solution I *(by Georg Osang)*

Take the points as vertex set,
and between the right ones let
there be a line of distance one.
Show this is plane and we are done:*



Pick any two of 1-distance;
as can be seen right at first glance,
others' distance square root three,
thus we're intersection-free.

*) *Haiku addendum:*
Recall, no more than
three times nodes subtracted six
edges in plane graphs.

Solution II *(by Fabian Stroh)*

I saw your problem written in verse,
thought: I am not challenge-averse.
I'll respond in rhyme,
one line at a time,
'Till I give up or my head hurts.

The first part that I have to show,
'bout the Graph you constructed so;
it's planar you see.
You can't disagree,
with the picture and common sense also.

(picture not pictured here)

To deal with just this situation
we know a corollary (for a triangulation)
of a formula by euler.
but it works just as well (spoiler)
for a less-dense-graphs stipulation.

The problem and solutions above were posed and submitted in the problem class of the course "Graph Theory" at the Karlsruhe Institute of Technology, winter semester 2011/2012, held by Prof. Maria Axenovich (http://www.math.kit.edu/iag/lehre/graphtheo2011w/).